1 (ii) $\tan^{-1}x + \tan^{-1}\frac{x}{1+1\cdot 2x^2} + \tan^{-1}\frac{x}{1+2\cdot 3x^2} + \dots + \tan^{-1}\frac{x}{1+n(n-1)x^2}$

B(H) Maths paper-1st, Gr. A summation of series Dr G.D. Singh

$$t_n = \tan^{-1}\frac{x}{1+nx\,(n-1)x}$$

$$= \tan^{-1}\frac{nx-(n-1)x}{1+nx\,(n-1)x}$$

$\tan^{-1}\frac{x-y}{1+xy} = \tan^{-1}x - \tan^{-1}y$

$$t_n = \tan^{-1}nx - \tan^{-1}(n-1)x$$

$$t_1 = \tan^{-1}x - \tan^{-1}0x$$

$$t_2 = \tan^{-1}2x - \tan^{-1}1x$$

$$t_3 = \tan^{-1}3x - \tan^{-1}2x$$

$$\dots$$

$$t_n = \tan^{-1}(nx) - \tan^{-1}(n-1)x$$

$$\therefore S = \tan^{-1}nx - \tan^{-1}0$$

$$= \tan^{-1}nx - 0 = \tan^{-1}nx \quad \underline{\text{Ans}}$$

1 (iv) $\tan^{-1}\frac{1}{2\cdot 1^2} + \tan^{-1}\frac{1}{2\cdot 2^2} + \tan^{-1}\frac{1}{2\cdot 3^2} + \dots + \tan^{-1}\frac{1}{2n^2}$

Here $t_n = \tan^{-1}\frac{1}{2n^2} = \tan^{-1}\frac{2}{4n^2}$

$$= \tan^{-1}\frac{2}{4n^2-1+1}$$

$$= \tan^{-1}\frac{2}{1+(2n+1)(2n-1)}$$

$$= \tan^{-1}\frac{(2n+1)-(2n-1)}{1+(2n+1)(2n-1)}$$

$$t_n = \tan^{-1}(2n+1) - \tan^{-1}(2n-1)$$

$$\therefore t_1 = \tan^{-1}3 - \tan^{-1}1$$

$$t_2 = \tan^{-1}5 - \tan^{-1}3$$

$$t_3 = \tan^{-1}7 - \tan^{-1}5$$

$$\dots$$

$$t_n = \tan^{-1}(2n+1) - \tan^{-1}(2n-1)$$

Adding $S = \tan^{-1}(2n+1) - \tan^{-1}1$

or $S = \tan^{-1} \dfrac{2n+1-1}{1+(2n+1)\cdot 1}$

$= \tan^{-1} \dfrac{2n}{2n+1+1} = \tan^{-1} \dfrac{2n}{2n+2}$

$= \tan^{-1} \dfrac{\cancel{2}n}{\cancel{2}(n+1)}$ Ans

Deduce to infinity.

Now $S_n = \tan^{-1} \dfrac{n}{n+1}$

$\therefore S_\infty = \lim_{n\to\infty} \tan^{-1} \dfrac{\frac{n}{n}}{1+\frac{1}{n}}$

$= \lim_{n\to\infty} \tan^{-1} \dfrac{1}{1+\frac{1}{n}}$

$= \tan^{-1} \dfrac{1}{1+0} = \tan^{-1} 1 = \pi/4$ Ans

Q. Sum to

$\cot^{-1}(2\cdot 1^2) + \cot^{-1}(2\cdot 2^2) + \cdots + \cot^{-1}(2\cdot n^2)$

Here $t_n = \cot^{-1}(2\cdot n^2) = \tan^{-1} \dfrac{1}{2n^2}$

इस उपर को याद करेगा। यही प्रश्न हो गया!

1.(v) $\tan^{-1}\frac{2}{4} + \tan^{-1}\frac{2}{9} + \tan^{-1}\frac{2}{16} + \cdots + \tan^{-1}\frac{2}{(n+1)^2}$

Here $t_n = \tan^{-1} \dfrac{2}{(n+1)^2}$

$= \tan^{-1} \dfrac{2}{1+2n+n^2} = \tan^{-1} \dfrac{2}{1+n(n+2)}$

$t_n = \tan^{-1} \dfrac{(n+2)-n}{1+n(n+2)} = \tan^{-1}(n+2) - \tan^{-1} n$

$\therefore t_n = \tan^{-1}(n+2) - \tan^{-1} n$

$\therefore t_1 = \tan^{-1}3 - \tan^{-1}1$

$t_2 = \tan^{-1}4 - \tan^{-1}2$

$t_3 = \tan^{-1}5 - \tan^{-1}3$

$t_4 = \tan^{-1}6 - \tan^{-1}4$

$t_5 = \tan^{-1}7 - \tan^{-1}5$

— — — —

$t_{n-2} = \tan^{-1}n - \tan^{-1}(n-2)$

$t_{n-1} = \tan^{-1}(n+1) - \tan^{-1}(n-1)$

$t_n = \tan^{-1}(n+2) - \tan^{-1}n$

Adding $S = \tan^{-1}(n+2) - \tan^{-1}(n+1) - \tan^{-1}2 - \tan^{-1}1$ Ans

1. (vii) Here $t_n = \cot^{-1}(1+n+n^2)$

$$= \tan^{-1}\frac{1}{1+n+n^2} = \tan^{-1}\frac{1}{1+n(n+1)}$$

अब w.o.1 (i) देखो वही प्रश्न हो गया।

2. (i) $\tan x \tan(x+y) + \tan(x+y)\tan(x+2y) + \dots$ to n terms

Now $\tan y = \tan\{(x+y)-x\}$

or $\tan y = \dfrac{\tan(x+y) - \tan x}{1+\tan(x+y)\tan x}$

$\cot y = \dfrac{1+\tan(x+y)\tan x}{\tan(x+y) - \tan x}$

or $1+\tan(x+y)\tan x = \cot y\{\tan(x+y) - \tan x\}$

or $\tan x \tan(x+y) = \cot y\{\tan(x+y) - \tan x\} - 1$

$\tan(x+y)\tan(x+2y) = \cot y\{\tan(x+2y) - \tan(x+y)\} - 1$

$\tan(x+2y)\tan(x+3y) = \cot y\{\tan(x+3y) - \tan(x+2y)\} - 1$

$\therefore$ जोड़ने पर $S = \cot y\{\tan(x+ny) - \tan x\} - n$ Ans